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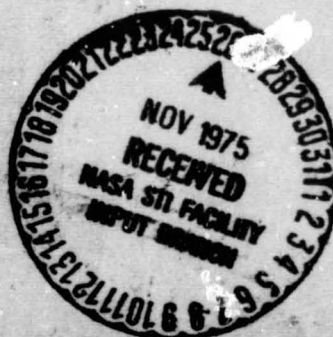
# LEHIGH UNIVERSITY

## STRESS ANALYSIS OF ADHESIVE BONDED STIFFENER PLATES AND DOUBLE JOINTS

BY

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## LIST OF SYMBOLS

$N_x^i$	membrane stress resultant in $i$ th plate
$Q_x^i$	shearing stress resultant in $i$ th plate
$M_x^i$	bending moment in $i$ th plate
$u^i$	in-plane displacement in $i$ th plate
$\beta^i$	rotation of normal in $i$ th plate
$w^i$	transverse displacement in the $i$ th plate
$C_{11}^i$	extensional stiffness of $i$ th plate
$D_{11}^i$	bending stiffness of $i$ th plate
$L_1^i$	transverse shearing stiffness of $i$ th plate
$h_i$	thickness of $i$ th plate
$t_{kj}$	thickness of adhesive layer between plate $k$ and $j$ , ( $k = 1, 2$ ; $j = k + 1$ )

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## ABSTRACT

The general problem of adhesive bonded stiffener plates and double joints of dissimilar orthotropic adherends with transverse shear deformations are analyzed. The adhesive layers are assumed to be of an isotropic, elastic and relatively flexible material. It is shown that the stress distributions in the adhesive layers are very much dependent on the bending deformations in adherends. Also, it is found that, in the adhesive layer, maximum transverse normal stress is, in many cases, larger than the longitudinal shear stress and that both occur at the edge of the joint. The general method of solution developed is applied to several practical examples.

## 1. INTRODUCTION

Recent advances in composite structures and the new adhesive bonding techniques based on very strong epoxy type adhesives have made feasible the adhesive joining and stiffening of structural elements subjected to extreme environmental and loading conditions. This type of joining and stiffening have been used extensively in flight and space vehicle structures. Epoxy based adhesives are also being used increasingly in stiffening, joining and repairing precast prestressed concrete and other structures.



Consequently, the importance of adhesive bonding in technology has been recognized and considerable amount of experimental and theoretical research has been carried out on adhesive bonded stiffener plates and double joints. Previous analytical work on stiffener plates can be found in papers by Muki and Sternberg [1], Cohen and De Silva [2], Erdogan [3], and Erdogan and Civelek [4]. The problems of double lap joint and double strap joint are considered by Lerchenthal [5], Volkersen [6], Szepe [7] and Segerlind [8]. Some experimental results are given by Mylonas [9]. Many similar problems and practical aspects of adhesives and bonding are also discussed in a book by Bickerman [10].

In many of the above cited references, either the bending deformations of the adherends (or, at least, of one adherend) are not taken into account or the solutions given are valid only for special cases of geometry, material, and loading conditions. The purpose of this paper, therefore, is to investigate the transverse normal stress and longitudinal shear stress distributions in the adhesive layers of bonded stiffener plates, double lap joints, and strap joints of dissimilar orthotropic adherends taking into account the bending of adherends. Furthermore, the transverse shear deformations of the adherends will also be included in the analytical model.

## 2. FORMULATION OF THE PROBLEM

In many practical applications, the adherends and the adhesive layers are arranged so that adherends are plate-like structures sandwiching one or more adhesive layers. Examples of such joints are the stiffener plate, double lap joint and strap joints of which simplest forms are shown in Figure 1.

In the following, the formulation of the problem of stiffener plate and double joints are practically the same except that the number of equations and the choice of boundary conditions change depending on the particular problem. Thus, a typical problem whether it be a stiffener plate or double joint or strap joint of length  $L$  is composed of upper, lower and middle adherends (or plates), with different orthotropic material constants and thicknesses  $h_1, h_2, h_3$ , and thin elastic, isotropic adhesive layers of thicknesses  $t_{12}, t_{23}$ . The principal directions of orthotropy in all adherends are assumed to coincide with the coordinate axes (see Figure 2). All the adherends are treated as plates subjected to in-plane tension, bending and thickness (or transverse) shear deformations. The adhesive layers are assumed to be relatively thin i.e.  $t_{kj} \ll h_i$  ( $i = 1, 2, 3$ ;  $k = 1, 2$ ;  $j = k + 1$ ) and they are assumed to behave elastically as simple tension-compression springs and shear springs, connecting the adherends. This implies that the variation of stresses through the adhesive layer is ignored and that only the transverse normal stress and longitudinal shear stress in the adhesive layer influence equilibrium.

The sign convention for displacements  $u^i, v^i, w^i$  ( $i = 1, 2, 3$ ), the strain quantities, the stresses and stress-resultants for both the adherends and the adhesive layers are those in Figure 2.

A more detailed treatment of the derivation of the equations of adhesive joints is presented in a previous report [11]. Assuming cylindrical bending in  $x$ -direction, the governing equations for the general bending problem of a stiffener plate (or double joint and strap joint, as the case may be) in the bonded portion are expressed by,

$$\frac{dN_x^i}{dx} = -p_x^i$$

$$\frac{dQ_x^i}{dx} = -p_z^i \quad (i = 1,2,3) \quad (1.a-c)$$

$$\frac{dM_x^i}{dx} = -m_x^i + Q_x^i$$

and,

$$\frac{du^i}{dx} = N_x^i / C_{11}^i$$

$$\frac{d\beta_x^i}{dx} = M_x^i / D_{11}^i \quad (i = 1,2,3) \quad (1.d-f)$$

$$\frac{dw^i}{dx} = Q_x^i / L_1^i - \beta_x^i$$

where single superscripts ( $i = 1,2,3$ ) indicate the particular adherend and  $N_x^i$ ,  $Q_x^i$ ,  $M_x^i$ ,  $u^i$ ,  $w^i$ ,  $\beta_x^i$  ( $i = 1,2,3$ ) are the fundamental variables, and  $p_x^i$ ,  $p_z^i$  ( $i = 1,2,3$ ) are the sum of the distributed surface loads and  $m_x^i$  ( $i = 1,2,3$ ) are the moments of these loads acting on the reference planes of the  $i$  th plate.

The load terms  $p_x^i$ ,  $p_z^i$ ,  $m_x^i$  ( $i = 1,2,3$ ), may be expressed as

$$\begin{aligned} p_x^1 &= q_x^1 - \tau^{12} \quad , \quad p_x^2 = \tau^{12} - \tau^{23} \quad , \quad p_x^3 = -q_x^3 + \tau^{23} \\ p_z^1 &= q_z^1 - \sigma^{12} \quad , \quad p_z^2 = \sigma^{12} - \sigma^{23} \quad , \quad p_z^3 = -q_z^3 + \sigma^{23} \end{aligned} \quad (2.a,b)$$

and,

$$m_x^1 = q_x^1 \frac{h_1}{2} + \tau^{12} \frac{h_1 + t_{12}}{2}$$

$$m_x^2 = \tau^{12} \frac{h_2 + t_{12}}{2} + \tau^{23} \frac{h_2 + t_{23}}{2} \quad (2.c-e)$$

$$m_x^3 = q_x^3 \frac{h_3}{2} + \tau^{23} \frac{h_3 + t_{23}}{2}$$

where single superscripts define a particular adherend and double superscripts define relative position of adhesive layer. Thus,  $q_x^1$ ,  $q_x^2$ , and  $q_z^1$ ,  $q_z^2$  are the distributed surface loads acting on adherends in x and z directions respectively. The quantities  $\tau$  and  $\sigma$  define the longitudinal shear stress in x- direction and transverse normal stress in z- direction in the adhesive (with superscripts indicating relative position of adhesive layer). Subscripts in geometric dimensions such as thickness h and t indicate the relative position of the corresponding adherend and adhesive layer, respectively.

The stresses in the adhesive layer (or layers) are obtained by writing the compatibility conditions (or the mechanical behavior of adhesive layers) along the bond length,

$$\begin{aligned} \sigma_z^{12}(x) &= \frac{B^{12}}{t_{12}} (w^1 - w^2) \\ \tau_x^{12}(x) &= \frac{G^{12}}{t_{12}} (u^1 - \beta_x^1 \frac{h_1}{2} - u^2 - \beta_x^2 \frac{h_2}{2}) \\ \sigma_z^{23}(x) &= \frac{B^{23}}{t_{23}} (w^2 - w^3) \\ \tau_x^{23}(x) &= \frac{G^{23}}{t_{23}} (u^2 - \beta_x^2 \frac{h_2}{2} - u^3 - \beta_x^3 \frac{h_3}{2}) \end{aligned} \quad (3.a-d)$$

where  $\sigma$ ,  $\tau$ , h, and t are defined previously, B is an elastic constant related to the Young's modulus and G is the shear modulus for a particular adhesive layer (superscripts of B and G again define the relative position of the adhesive layer between the adherends). The quantities u, w,

and  $\beta_x$  are displacements in x, z directions and the angle of rotation of the normal in z- direction respectively.

The elastic constants  $B^{kj}$  ( $k = 1,2; j = k + 1$ ) in adhesive layers can be expressed in terms of the Young's moduli  $E^{kj}$  ( $k = 1,2; j = k + 1$ ). For this purpose, consider the elastic stress-strain law in any adhesive layer "kj" in cylindrical bending (no summation on k and j),

$$\sigma_z^{kj} = \lambda^{kj} (e_x + e_z)^{kj} + 2 G^{kj} e_z^{kj} \quad (4)$$

where  $\sigma_z^{kj}$  is the normal stress and  $e_x^{kj}, e_y^{kj}, e_z^{kj}$  are the strains and  $\lambda^{kj}$  is the Lamé elastic constant given by

$$\lambda^{kj} = \nu^{kj} E^{kj} / [(1 + \nu^{kj})(1 - 2\nu^{kj})] \quad (5)$$

The compatibility of strains, on the interfaces between adherends and adhesive layers, requires that  $e_x^{kj}$  ( $k = 1,2; j = k + 1$ ) in any adhesive layer must be equal to the adherend strains  $e_x^i$  ( $i = 1,2,3$ ) on the interfaces, however  $e_z^{kj}$  of the adhesive layer given by,

$$e_z^{kj} = (w^k - w^j) / t_{kj} \quad (6)$$

can be much larger or with  $(t_{12}, t_{23}) \ll h_i$  ( $i = 1,2,3$ ), in the adhesive layer,

$$|e_z^{kj}| \gg |e_x^{kj}| \quad (7)$$

then, neglecting  $e_x^{kj}$  in comparison with  $e_z^{kj}$  in (4), the elastic constant  $B^{kj}$  of the adhesive layer becomes,

$$B^{kj} \cong \lambda^{kj} + 2G^{kj} = (1 - \nu^{kj})E^{kj} / [(1 + \nu^{kj})(1 - 2\nu^{kj})] \quad (8)$$

which means that  $B^{kj}$  is larger than  $E^{kj}$  in the adhesive.



The substitution of (3.a-d) into (2.a-e), and then replacing  $p_x^i$ ,  $p_z^i$ ,  $m_x^i$  ( $i = 1, 2, 3$ ) in (1.a-f), the system of equations with the appropriate boundary conditions finally reduce to a two-point boundary problem in the region of  $(-\ell \leq x \leq +\ell)$  and  $(-\infty \leq y \leq +\infty)$ ,

$$\frac{d}{dx} Y_r(k) = \sum_k A_{rk} Y_k(x) + P_r(x) \quad , \quad (r = k = 1, 2, \dots, 18) \quad (9.a)$$

and,

$$\begin{aligned} \sum_k T_{mk}^- Y_k(-\ell) &= U_{*m}^- \\ \sum_k T_{nk}^+ Y_k(+\ell) &= U_{*n}^+ \end{aligned} \quad (m = n = 1, 2, \dots, 9) \quad (9.b,c)$$

where  $A_{rk}$  is a coefficient matrix of order (18,18) including elastic constants and geometric dimensions such as thicknesses, etc. of adherends and adhesive layers [\*].  $P_r(x)$  is a column matrix of order 18, corresponding to the distributed surface loads  $q_x^i$ ,  $q_z^i$ . The matrix  $Y_r(x)$  is the column matrix of order 18, including all the fundamental variables of the three adherends as its elements. In the boundary conditions of (9.b,c), the matrices  $T_{mk}^-$ ,  $T_{nk}^+$  are constant matrices each of order (9,18) and, in general, are, depending on support conditions, unit matrices. The quantities  $U_{*m}^-$  and  $U_{*n}^+$  are column matrices corresponding stress resultants and displacements in the boundary conditions prescribed at  $x = -\ell$  and  $x = +\ell$  respectively.

The boundary conditions are obtained from known stress resultants and displacements of adherends at the ends of the joint. For instance, the

[\*] If the thicknesses of the adherends are not constant, then the coefficient matrix  $A_{rk}$  becomes a function of  $x$  i.e.  $A_{rk}(x)$ . However, this does not pose any additional difficulty in solving the system of (9.a-c) by the method employed in this work.



appropriate boundary conditions for the stiffener plate subjected to an external tension  $N_0$  and considering the symmetry of the system are as follows:

at  $x = 0$  the column matrix  $U_{*m}^-$ :

$$\begin{aligned} N_x^1 = 0, \quad Q_x^1 = 0, \quad M_x^1 = 0, \quad u^2 = 0, \quad w^2 = 0, \\ \beta_x^2 = 0, \quad N_x^3 = 0, \quad Q_x^3 = 0, \quad M_x^3 = 0 \end{aligned} \quad (10.a)$$

at  $x = +\ell$  the column matrix  $U_{*n}^+$ :

$$\begin{aligned} N_x^1 = 0, \quad Q_x^1 = 0, \quad M_x^1 = 0, \quad N_x^2 = N_*^2, \quad Q_x^2 = Q_*^2, \\ M_x^2 = M_*^2, \quad N_x^3 = 0, \quad Q_x^3 = 0, \quad M_x^3 = 0 \end{aligned} \quad (10.b)$$

where the subscripts "\*" designates quantities prescribed at  $x = 0, \ell$ . A more detailed discussion of the appropriate conditions is given in [11].

In general, the system of 18 ordinary differential equations in (9.a) and two sets of 9 boundary conditions in (9.b,c) constitutes a two-point boundary value problem of order 18.

### 3. SOLUTION OF THE SYSTEM OF EQUATIONS

In the case of dissimilar adherends with unequal but uniform thicknesses and different elastic properties, a multi-segment method of integration is the most suitable to solve the system in (9.a-c) (see, for instance, [11] and [12]).

In the practical case of geometric, material and external loading symmetry with respect to the  $x$ -axis, the number of equations in (1.a-f) reduces to 12 and the adhesive layer stress in (3.a-d) can be replaced by,

$$\sigma_z^{12}(x) = \sigma_z^{23}(x) + \sigma(x) \quad , \quad \tau_x^{12}(x) = \tau_x^{23}(x) + \tau(x) \quad (11.a,b)$$

$$h_1 = h_3 + h_1 \quad , \quad h_2 + h_2 \quad , \quad t_{12} = t_{23} + t$$

Thus, the solution of the two-point boundary value problem may be obtained numerically by the method described in [11]. This procedure is sufficiently general so as to allow for non-symmetry and the variation of adherend thickness and material properties along the joint [11, 12].

The effect of the average transverse shear deformations of adherends are given by the terms  $Q^i/L_1^i$  ( $i = 1,2,3$ ) in (1.a-f). The influence of these terms on the adhesive stresses  $\tau(x)$  and  $\sigma(x)$  were discussed in [11]. Therefore it will not be repeated here.

#### 4. DISCUSSION OF RESULTS

A computer program for the solution of the equations of stiffener plates and double joints bonded through one or more flexible adhesive layers has been developed. The numerical results for several typical problems are discussed in the following.

##### a. Stiffener Plate with Dissimilar Adherends

As an example, the adhesive stresses  $\tau(x)$  and  $\sigma(x)$  in the case of an aluminum plate stiffened by a single boron-epoxy plate subjected to the basic external loads  $N_0$ ,  $Q_0$ ,  $M_0$  are presented in Figures 3, 4, and 5, respectively. It can easily be seen that, especially in Figures 4 and 5,  $\sigma_{\max}$  is larger than  $\tau_{\max}$  when the external bending is dominant in the unstiffened section of the base plate.

In order to illustrate the significant effect of bending of adherends, even in the absence of external bending moment, the analytical model for the stiffener plate solution of this paper may be compared with that of Erdogan and Civelek [4]. The results of both methods corresponding to an external tension  $\sigma_0$  for the identical stiffener-base plate combination are plotted in Figure 6. The curves "a" and "b" are obtained by Erdogan and Civelek [4]. In their analysis, "a" is based on a model which treats the base plate as an elastic continuum and the stiffener plate as a membrane while "b" is based on both plates being membranes [4]. It is easy to see that, in Figure 6, the maximum adhesive shear stress  $\tau_{\max}$  calculated by the present method is somewhat less than the  $\tau_{\max}$  values of "a" and "b" obtained by Erdogan and Civelek [4]. In both cases "a" and "b", Erdogan and Civelek [4] assume that the transverse normal stress  $\sigma(x)$  in the adhesive is identically zero and also ignore the bending resistance of the stiffener plate.

It should also be emphasized here that in [4], only external uniform tension loading  $\sigma_0$  is taken into account as an external load acting on the base plate. If the same base plate is subjected to an external bending stress, it can be expected that  $\tau_{\max}$  and  $\sigma_{\max}$  in the adhesive are to be increased drastically. This, in fact, is the case seen in Figure 7. The adhesive stresses  $\tau(x)$  and  $\sigma(x)$  corresponding to an identical stiffener-plate combination subjected to an external bending stress  $\sigma_\star^2$  which is equal to the average tension  $\sigma_0$  of [4] in the unstiffened section are calculated by the present method and plotted in Figure 7. A simple comparison of  $\tau_{\max}$  values for the same geometry and material in Figures 6 and 7 indicates that  $\tau_{\max}$  due to an external bending moment producing the

same nominal stress  $\sigma_0$  is larger than  $\tau_{\max}$  due to the uniform external tension  $\sigma_0$ . Not only that but also there is an additional stress that is the transverse normal stress  $\sigma_{\max}$ , in Figure 7, which is very much larger than  $\sigma_{\max}$  of Figure 5. Thus, it is obvious that the normal stress  $\sigma(x)$  in the adhesive layer and the effect of bending of adherends on the adhesive stresses  $\tau(x)$  and  $\sigma(x)$ , specifically in the latter, cannot be ignored as it was done in [4] in the analysis of stiffener-plate combinations when external loads are bending moments.

#### b. Double Lap Joint with Dissimilar Adherends

The distribution of adhesive stresses in a typical lap joint composed of two aluminum plates bonded to a boron-epoxy plate subjected to an external tension load  $N_0$  is calculated and plotted in Figure 8. It should be noted here that, due to the symmetry of the geometry and the loading with respect to x- axis, there is no bending in the middle adherend. The calculated values of  $\sigma_{\max}$  and  $\tau_{\max}$  are of the same order of magnitude. However, if the joint is under the action of external shear loads or bending moments it can be expected that  $\sigma_{\max}$  could be the dominant stress in the adhesive layer.

In this connection, a variant of the so called the "trouser leg" problem for the same double lap joint configuration is also considered. This double lap joint configuration is subjected to self-equilibrating external shear  $Q_0$ , external moment  $M_0$  and the results are plotted in Figure 9 and Figure 10 respectively. It can be seen that, in both cases of the trouser leg problem, the  $\sigma_{\max}$  stresses are very much larger than that of the double lap joint of identical geometry and material under only external

tension  $N_0$  in Figure 8. Even more so in the case of external bending moment  $M_0$  given in Figure 10.

c. Single Strap and Double Strap Joints with Dissimilar Adherends

The stress analysis of a single strap joint of boron-epoxy bonded through an epoxy layer to two aluminum plates subjected to basic external loadings is considered in Figures 11, 12, and 13. It is obvious that, even in the external tension case, the transverse normal stress  $\sigma_{\max}$  is very much larger than the longitudinal shear stress  $\tau_{\max}$  and that the larger stress concentrations occur as expected at the point of geometric and material discontinuity that is at the middle of the jointing plate.

As a further example a double strap joint is also analyzed (see Figure 12). It is of interest to observe here that, in contrast to the single strap joint, the stresses  $\sigma_{\max}$  and  $\tau_{\max}$  are of the same order of magnitude. This is of course not unexpected since the symmetry of the loading and geometry as in Figure 14 prevent the bending deformations in the middle adherends. In this respect, the similarity of the stress distributions in the double strap joint in Figure 14 and the double lap joint in Figure 8 should be noted.

## 5. CONCLUSIONS

1. Concentrations of stresses  $\tau_{\max}$  and  $\sigma_{\max}$  in the adhesive layer occur at locations of geometric and material discontinuity namely, at the ends of the joint.



2. The present analytical model does not take into account the fact that the adhesive shear stress  $\tau(x)$ , after reaching to peak values in a small distance from both edges, ought to be zero at both ends of the joint. This point was considered for only double lap joints by Volkersen [6]. However, his solution is valid only for a special case of isotropic, symmetrical adherends of identical material with an upper (or lower) adherend to middle adherend thickness ratio equal to 1/2.

3. Bending deformations of adherends, even if only in one of the adherends, have a significant effect on  $\tau(x)$  and  $\sigma(x)$ , particularly in the latter, in the adhesive layer. This bending effect is observed in stiffener plate and double lap joint problems, even in the case of direct external tension loading.

4. Due to this bending effect, the maximum normal stress  $\sigma_{\max}$  can be larger than the maximum shearing stress  $\tau_{\max}$  in the adhesive layer.

5. The solution technique employed in this paper can easily be used for problems of double joints and stiffener plates with tapered adherends. The effect of adherend transverse shear deformation may be included without difficulty.



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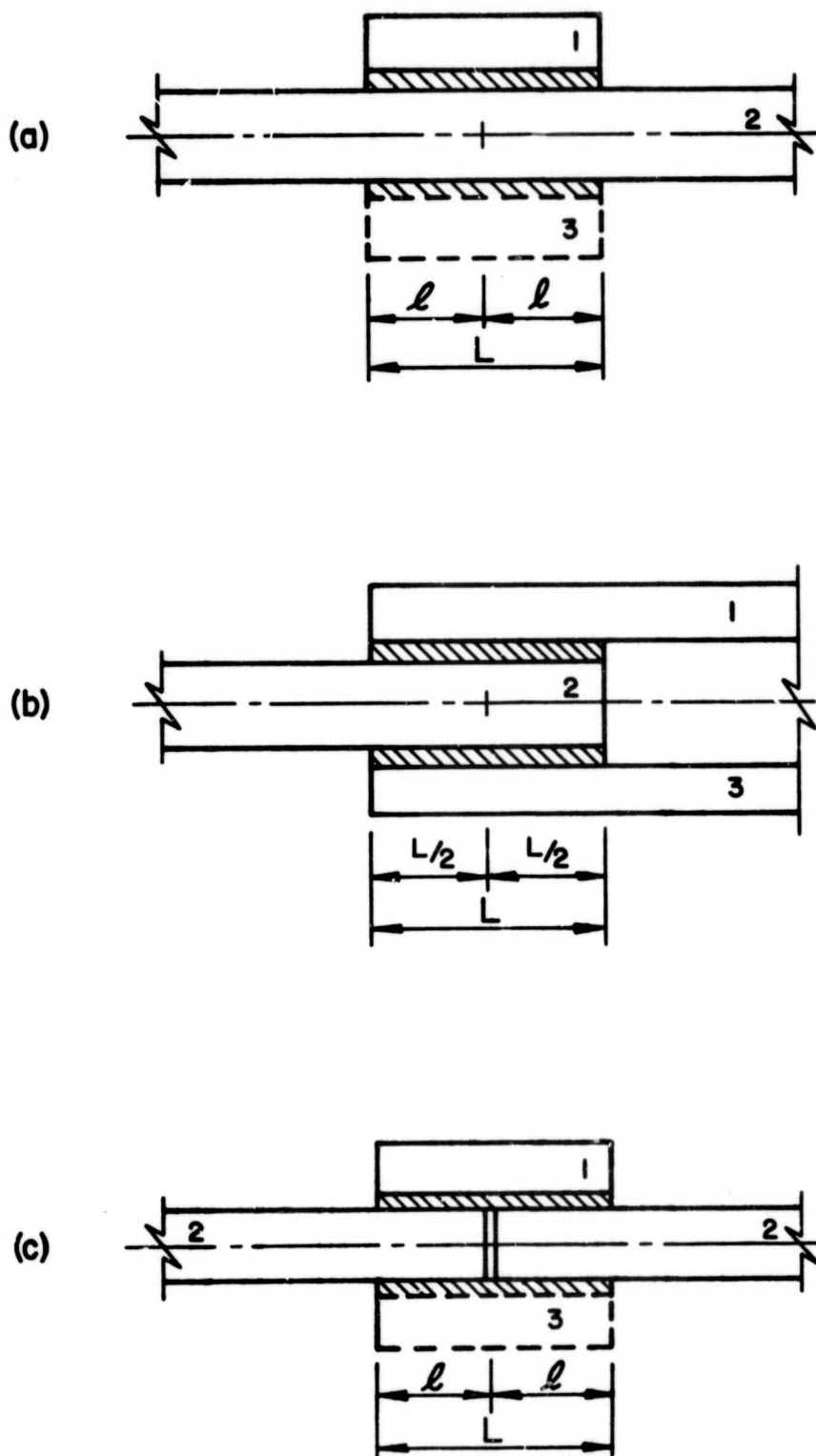


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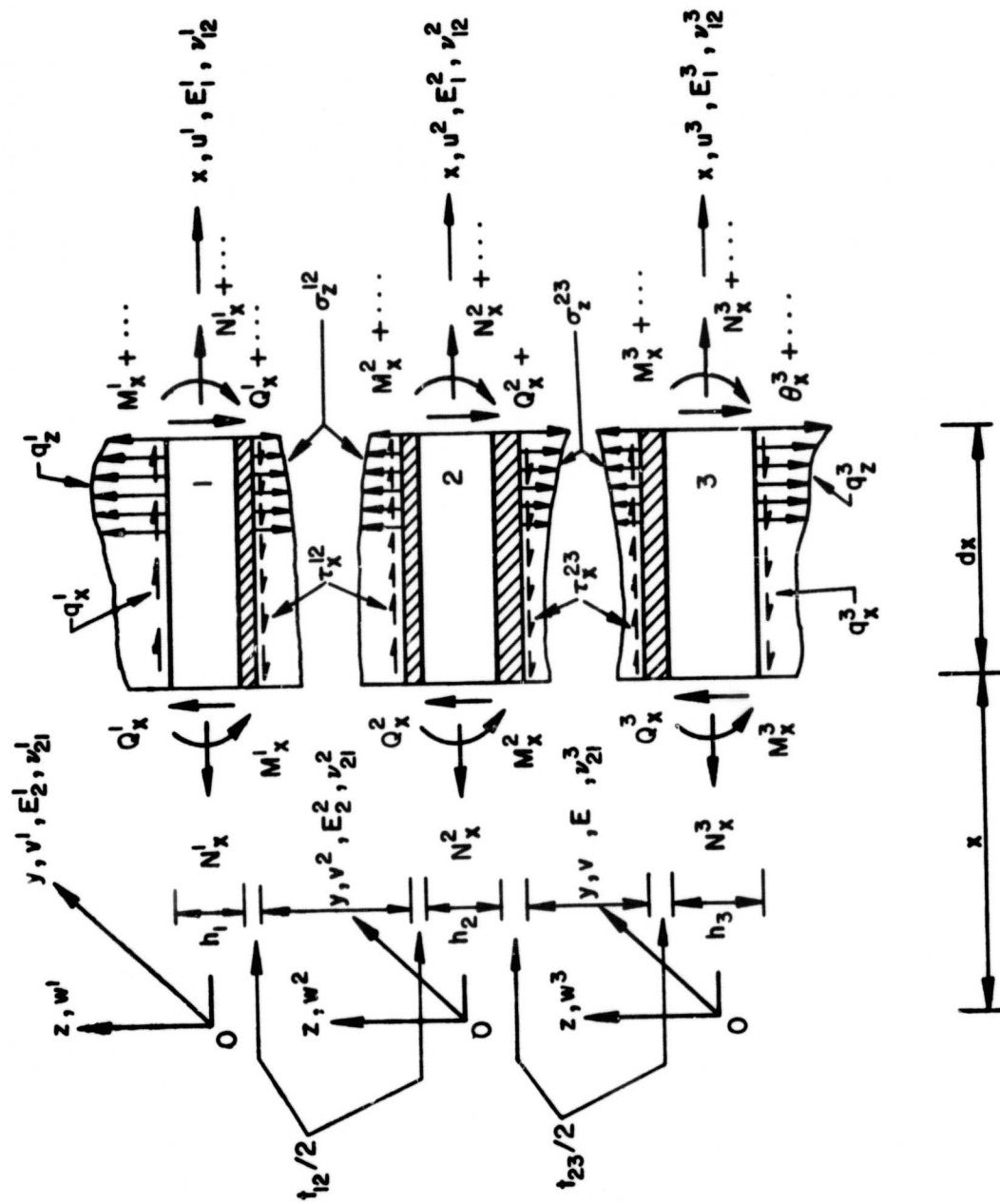


Figure 2 Coordinate System and Sign Convention in the Equilibrium Element (Cylindrical Bending)

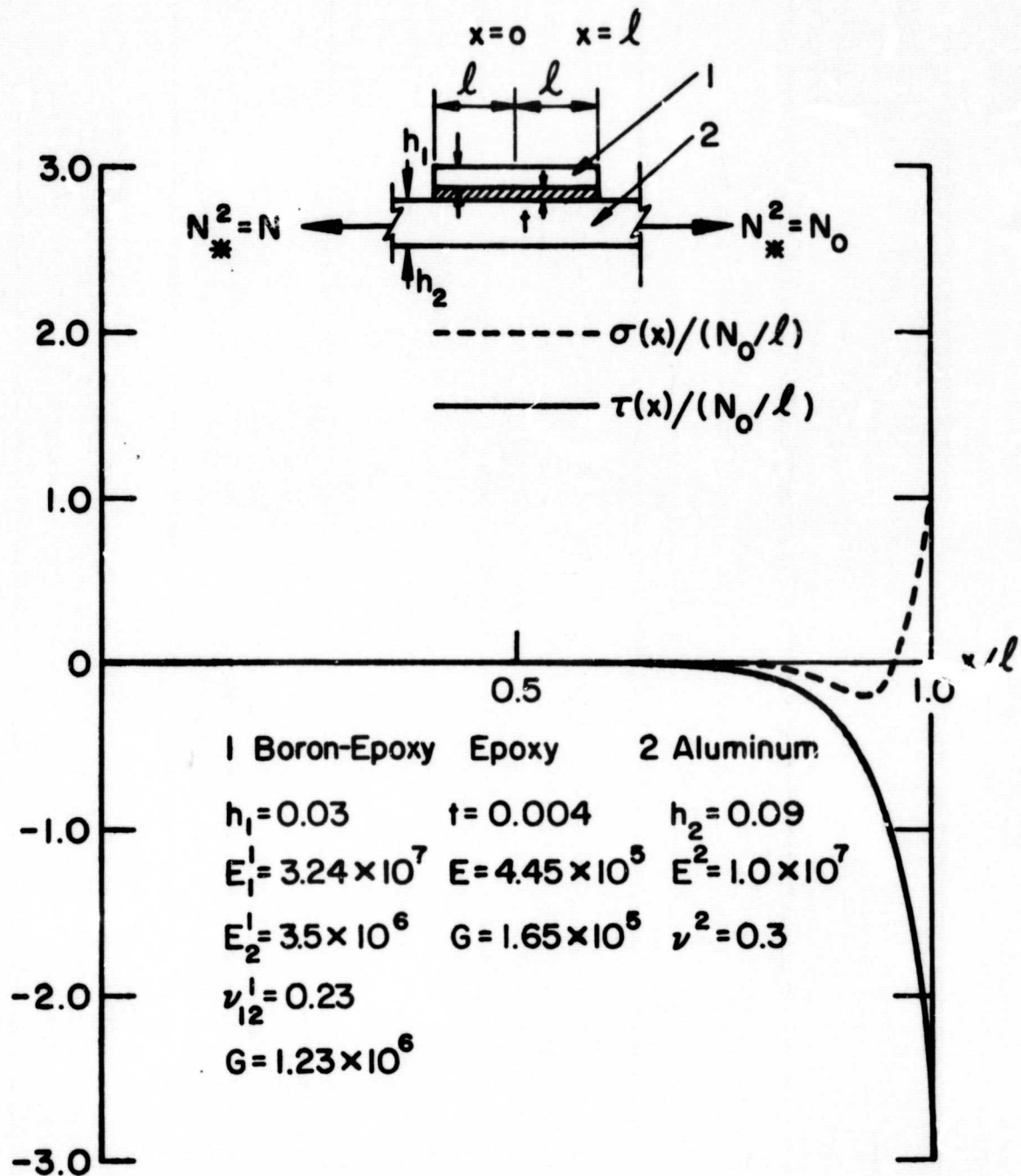


Figure 3 Stiffener Plate with Dissimilar Adherends  
-External Tension Loading  $N_0$



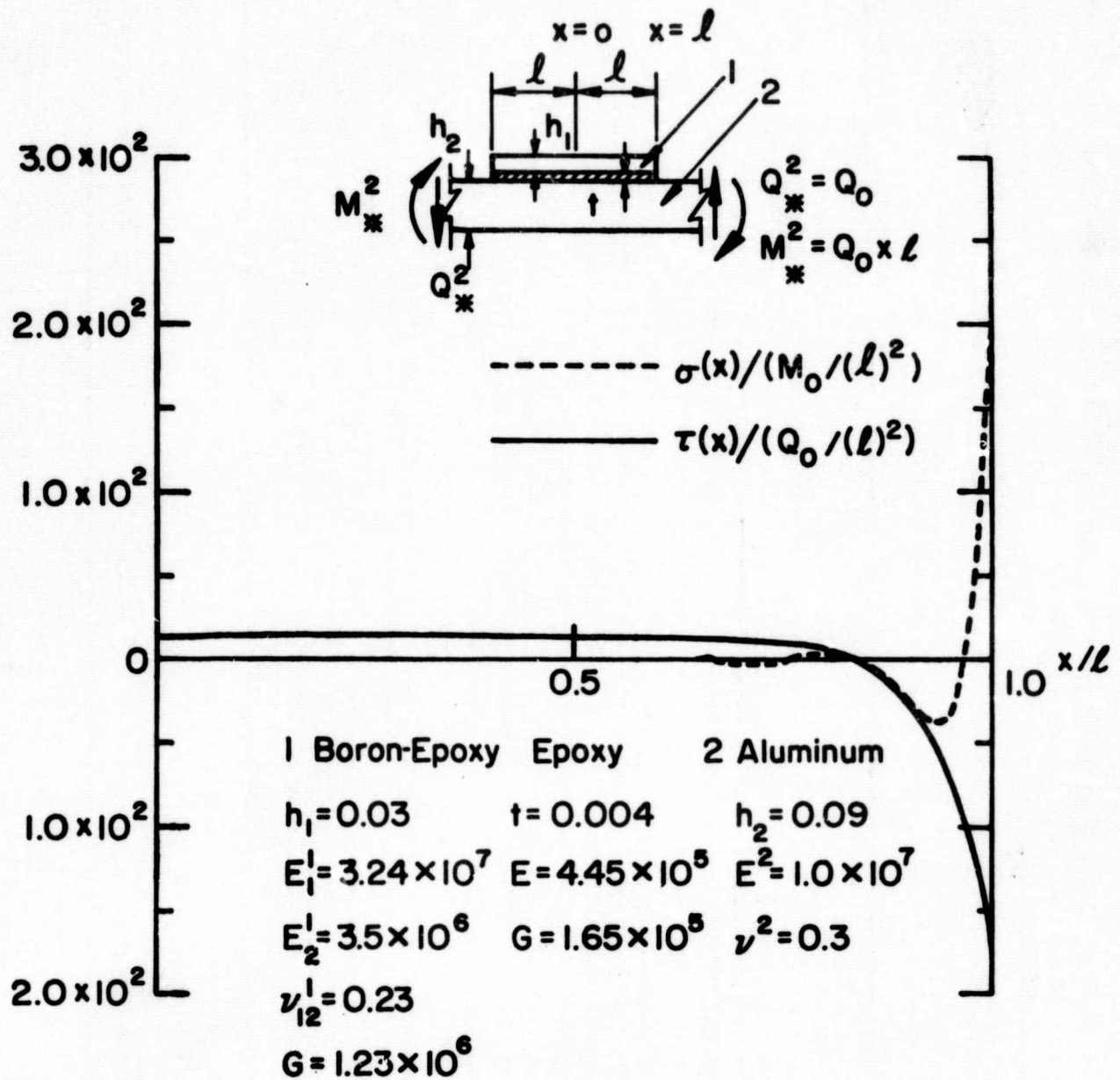
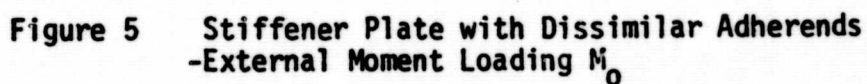


Figure 4 Stiffener Plate with Dissimilar Adherends  
-External Transverse Shear Loading  $Q_0$



**Figure 5 Stiffener Plate with Dissimilar Adherends  
-External Moment Loading  $M_0$**

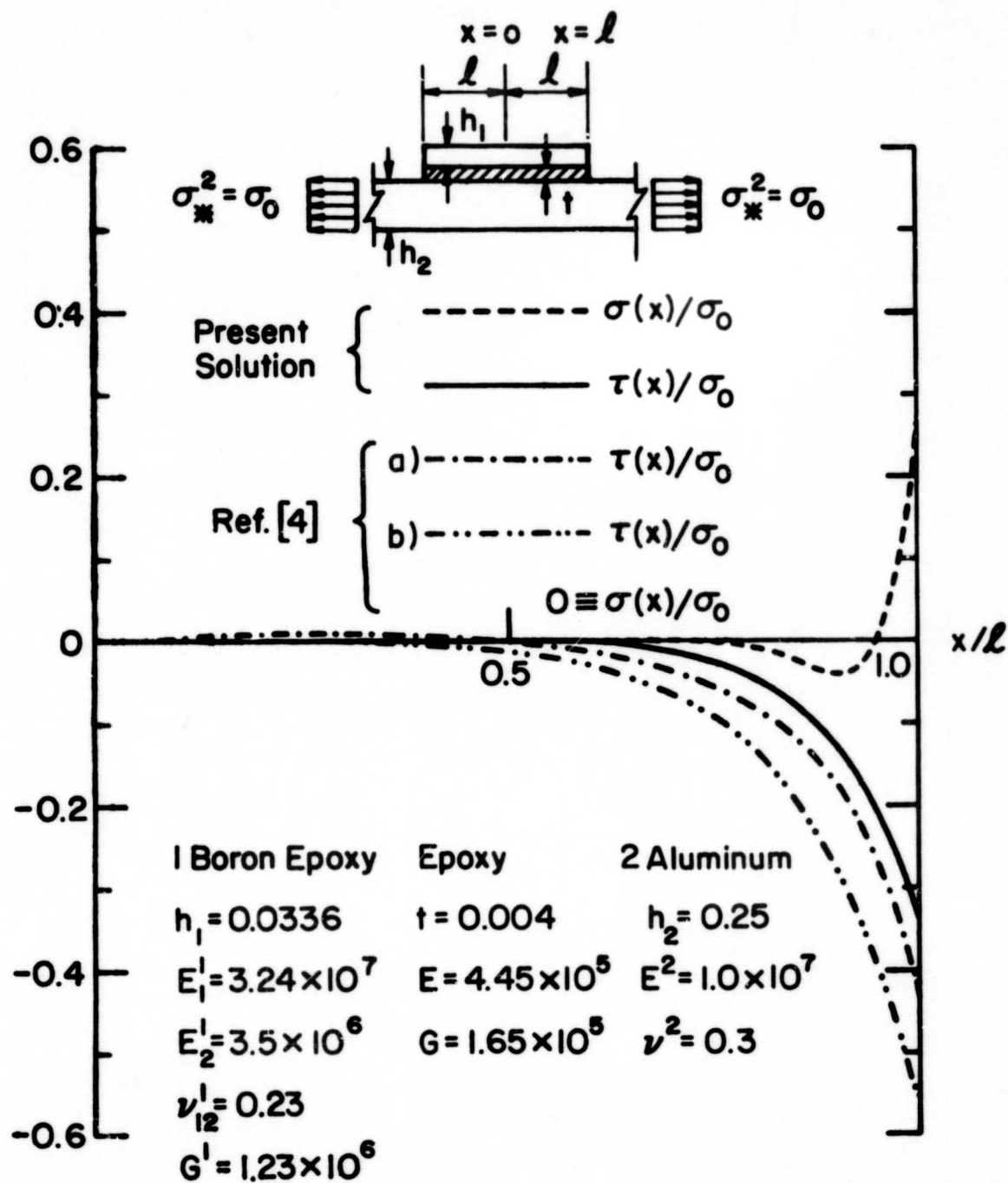


Figure 6 Bending Effect in a Stiffened Section  
-Direct Tensile Stress  $\sigma_0$

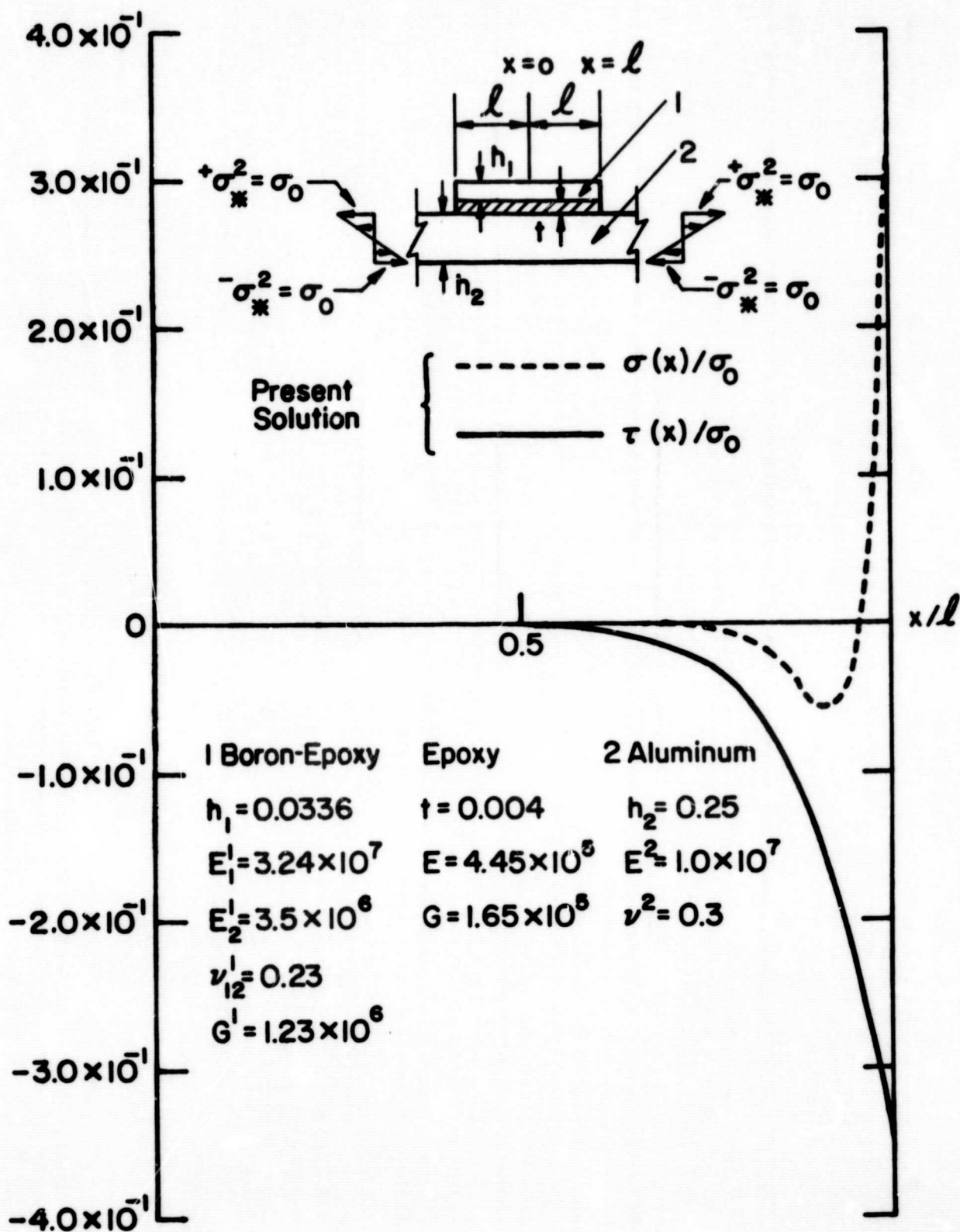


Figure 7 Bending Effect in a Stiffened Section  
-External Bending Stress  $\sigma_0$

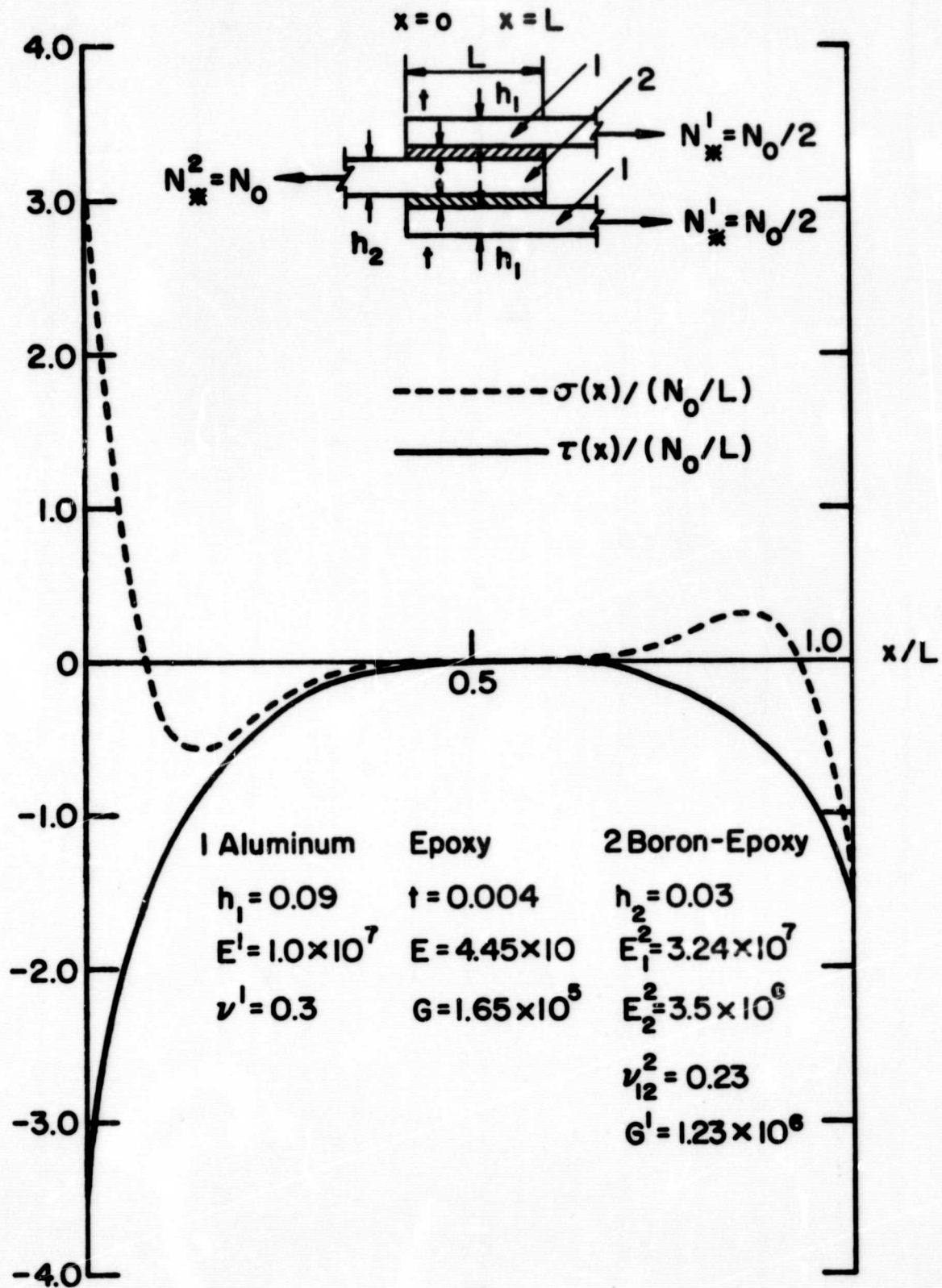


Figure 8 Symmetric Double Lap Joint with Dissimilar Adherends  
-External Tension Loading  $N_0$



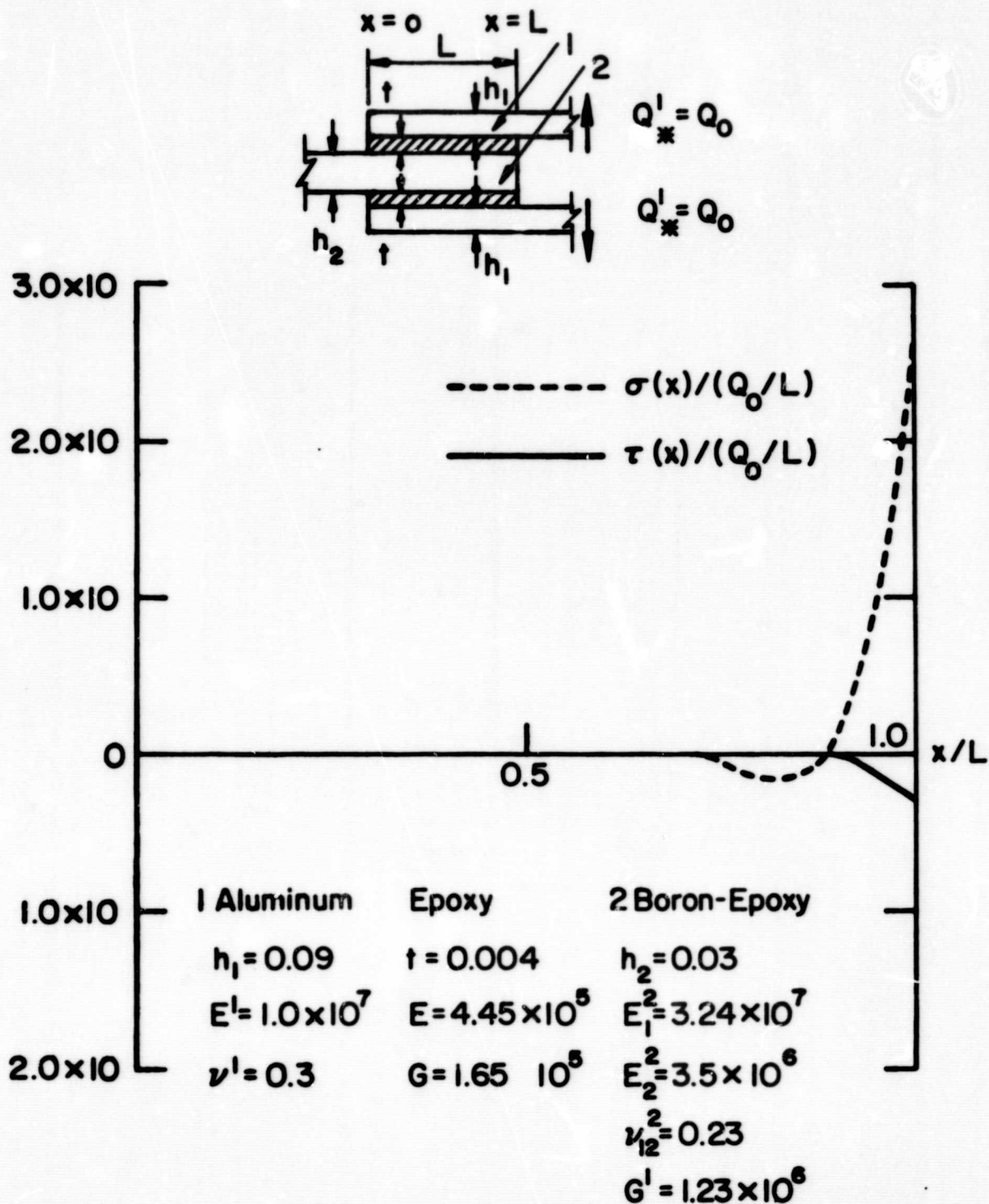


Figure 9 A Variant of the "Trouser Leg" Problem  
-External Shear Loading  $Q_0$



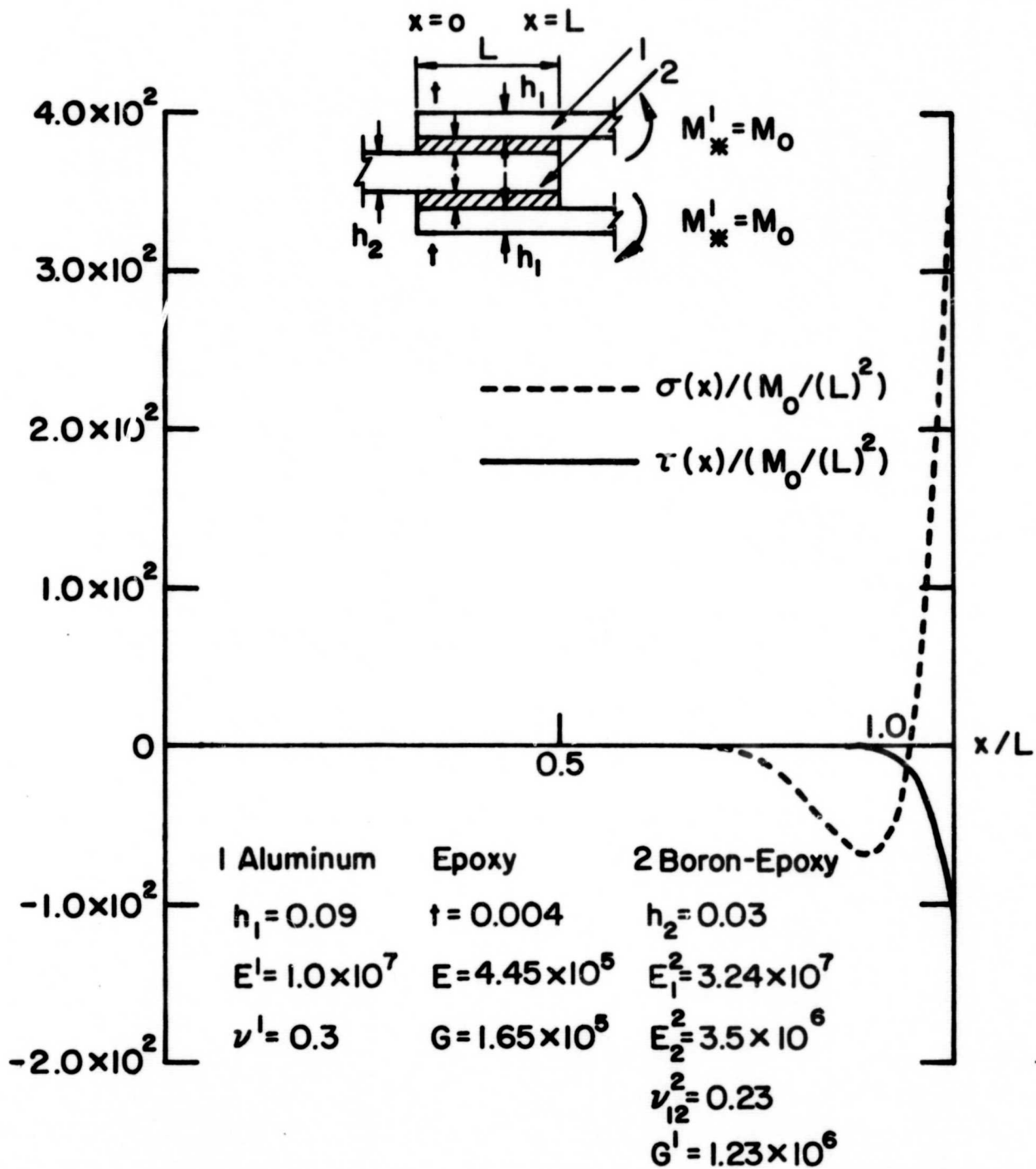


Figure 10 A Variant of the "Trouser Leg" Problem  
-External Bending Moment  $M_0$

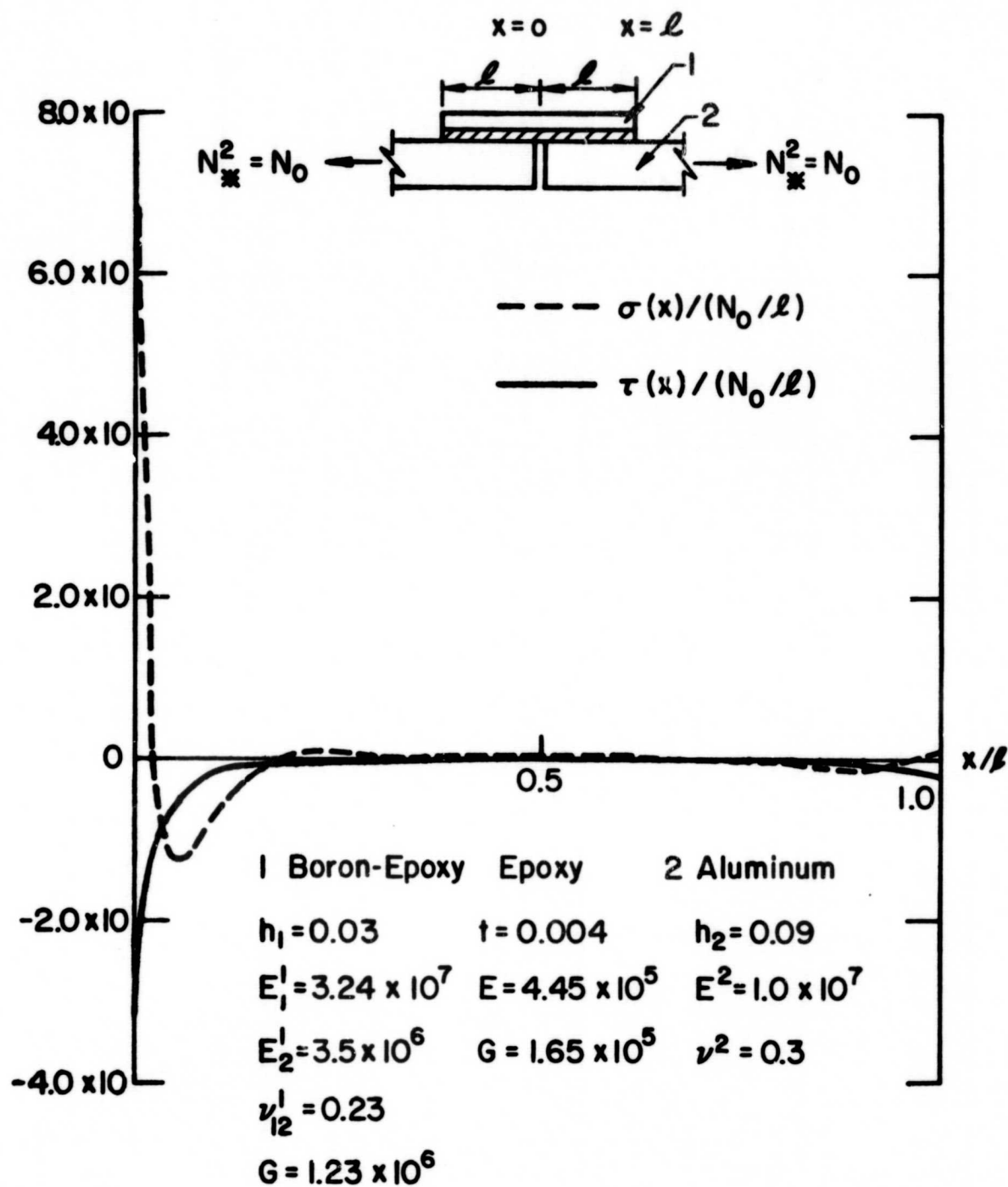


Figure 11 Strap Joint with Dissimilar Adherends  
 -External Tension Loading  $N_0$

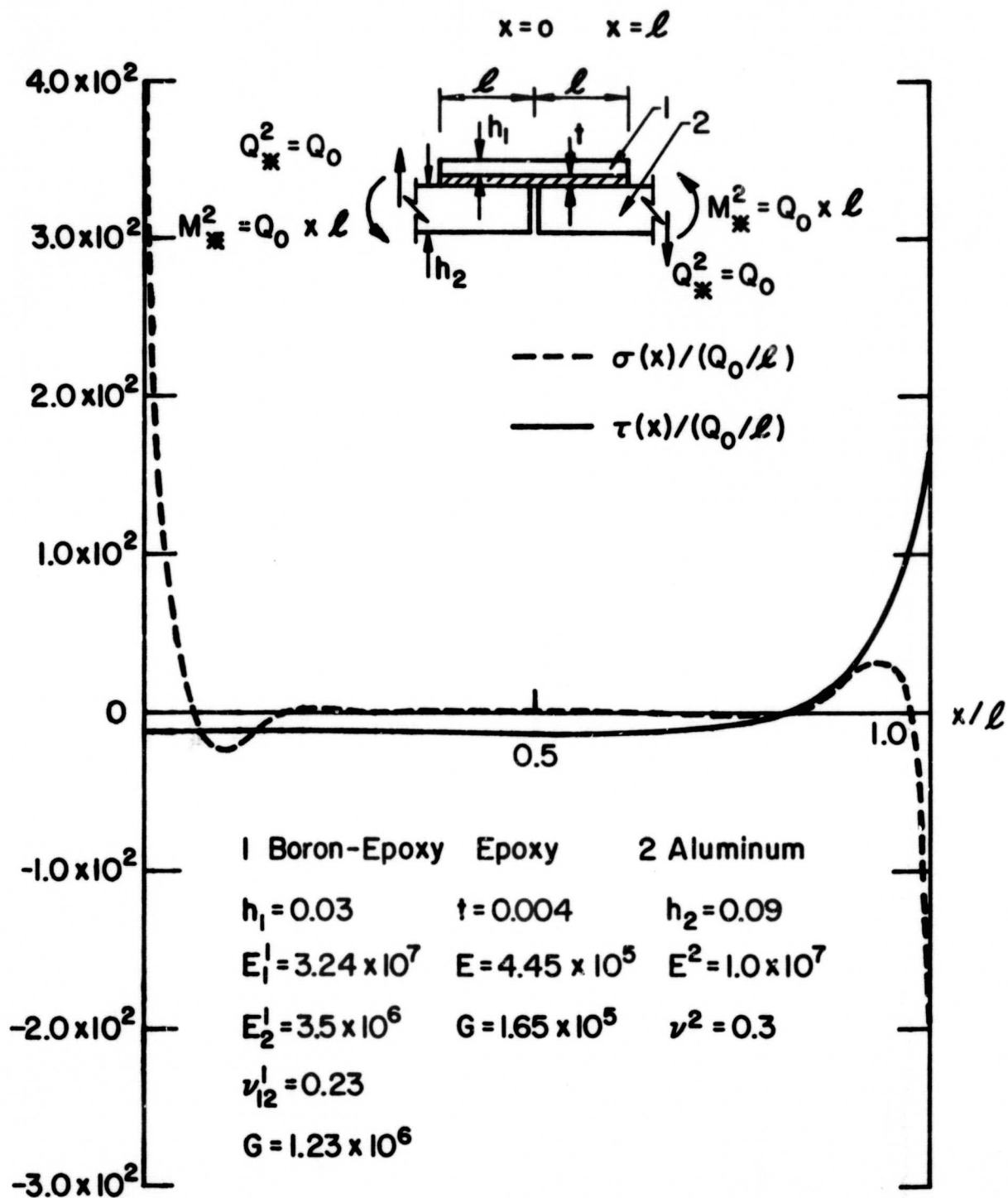


Figure 12 Strap Joint with Dissimilar Adherends  
-Transverse Shear Loading  $Q_0$

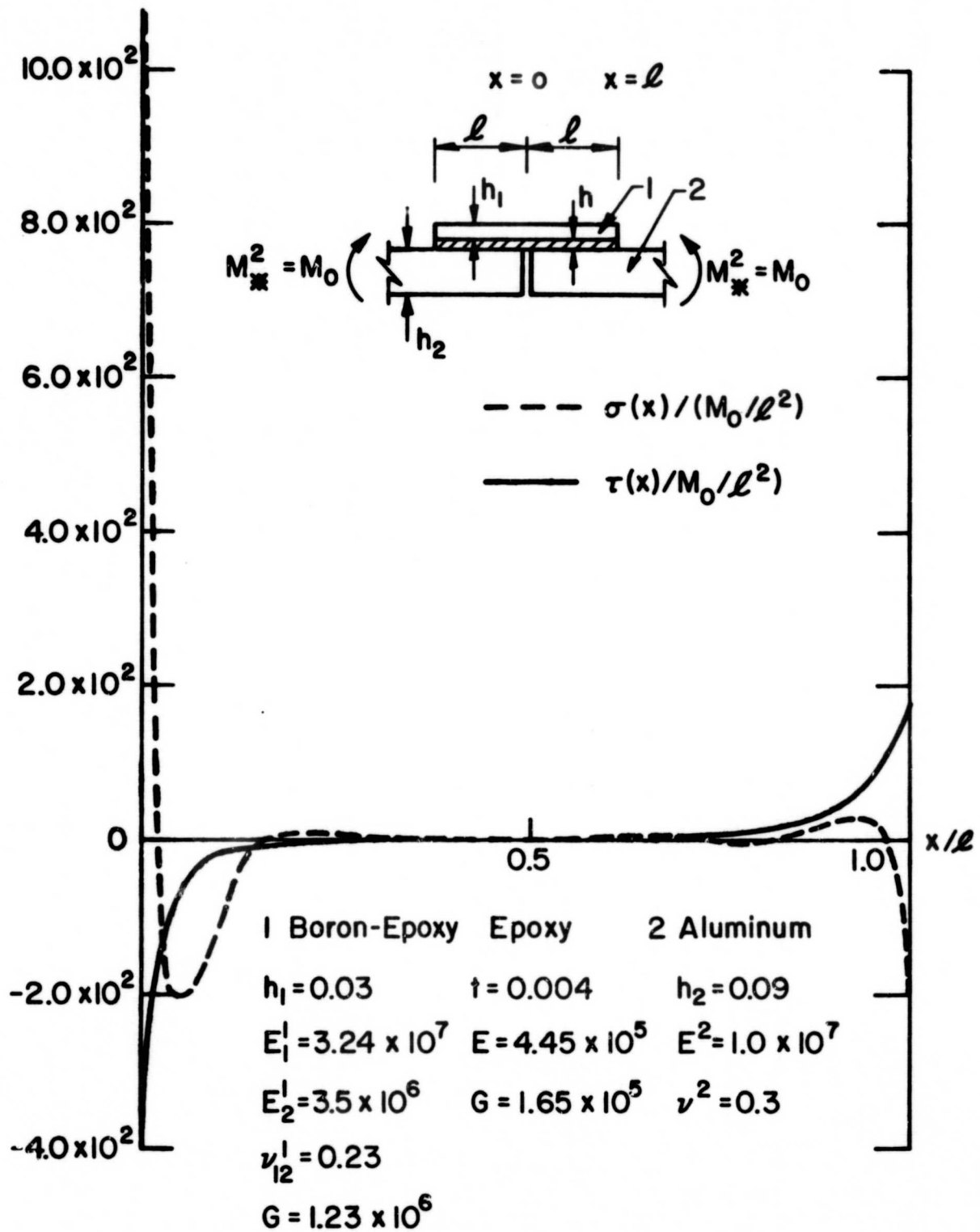


Figure 13 Strap Joint with Dissimilar Adherends  
-External Moment Loading  $M_0$

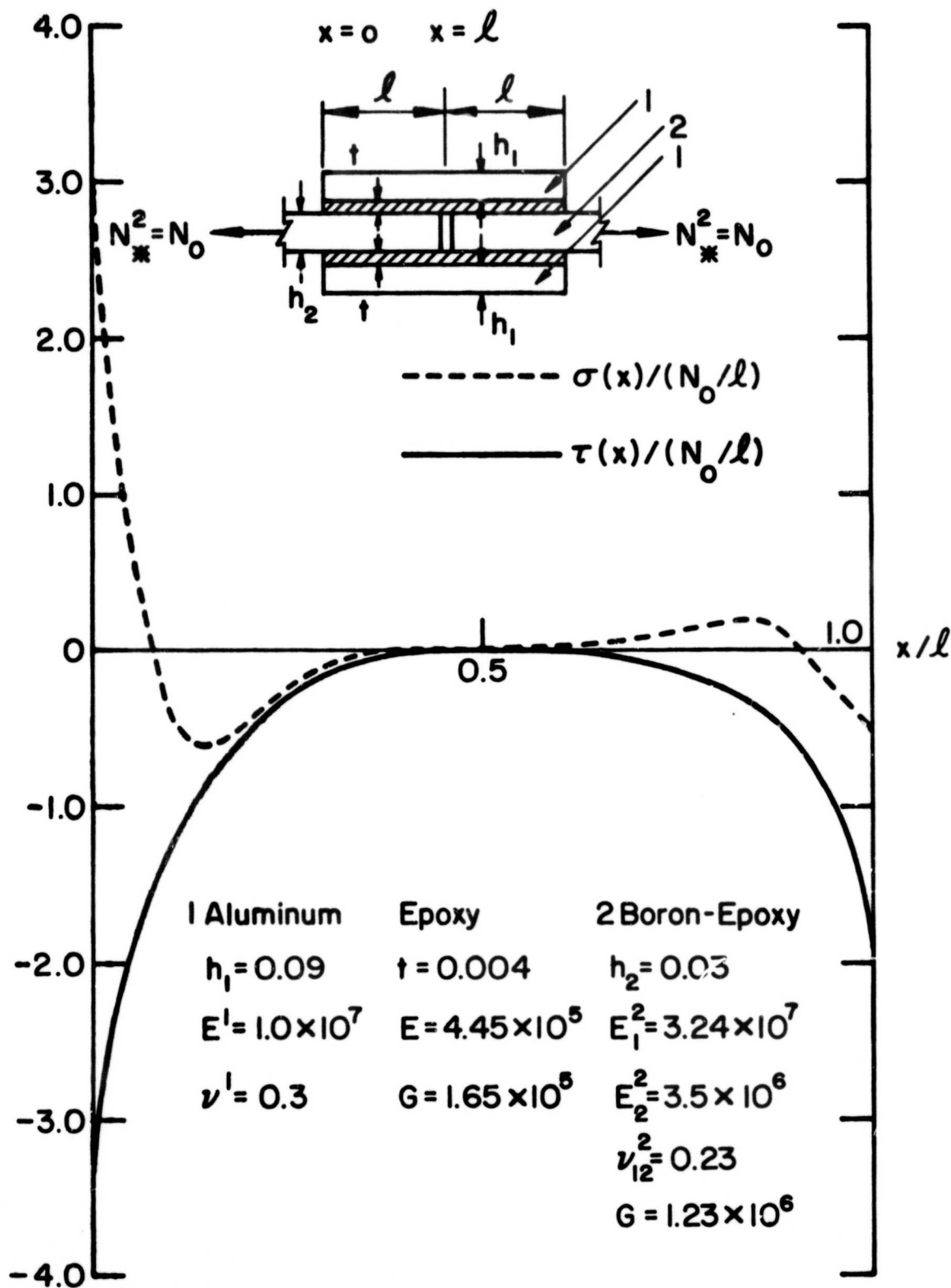


Figure 14 Symmetric Double Strap Joint  
-External Tension Loading  $N_0$